MCC121 Microwave Engineering
3D Electromagnetic simulations of waveguide components

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Field propagation in rectangular waveguide

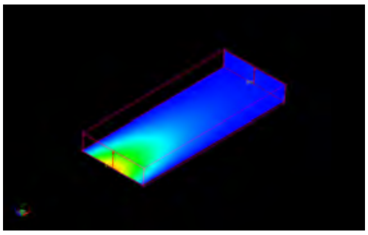
This first task is about simulating an electromagnetic field in a rectangular waveguide.

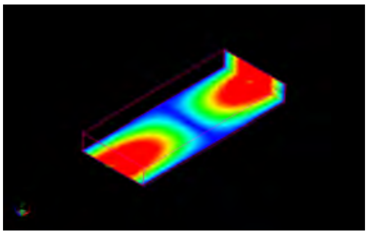
Task A

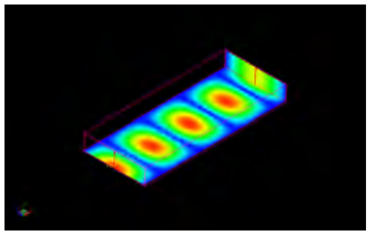
In this subtask, the TE_{10} mode was going to be simulated in a rectangular waveguide with the dimensions 40 mm $\times$ 10 mm $(a \times b)$. The simulations was supposed to be done for the frequencies ranging between 4 GHz and 9 GHz. By using the well known formula for the cutoff frequency of a rectangular waveguide,

$$f_{c_{mn}} = \frac{c}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}, \tag{1}$$

where m and n are the indexing of the modes, it can be obtained that the cutoff frequency for the given waveguide at TE_{10} is $f_{c_{10}} = 3.75$ GHz. This implies that one has to simulate for frequencies lower than stated in the description to get some simulation data of interest. In Figure 1 the E-field in the waveguide



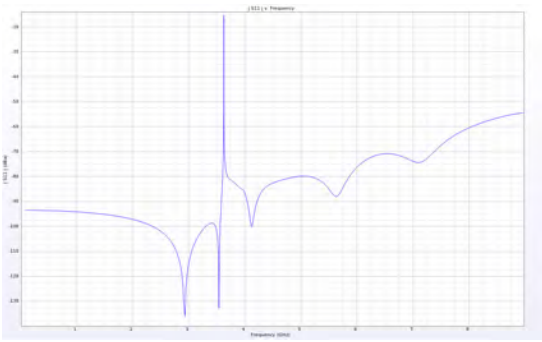


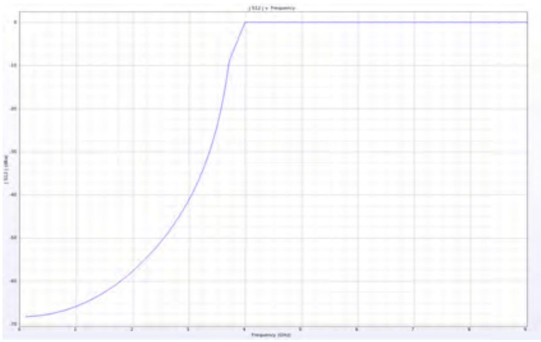


(a) E-field at 3.3 GHz. (b) E-field at 4 GHz. (c) E-field at 7 GHz.

Figure 1: E-field propagation in the given waveguide.

for different frequencies can be seen. As can be observed, there is no propagating wave until a frequency of 4 GHz is reached. The S-parameters of the waveguide can be seen in Figure 2. In Figure 2a the





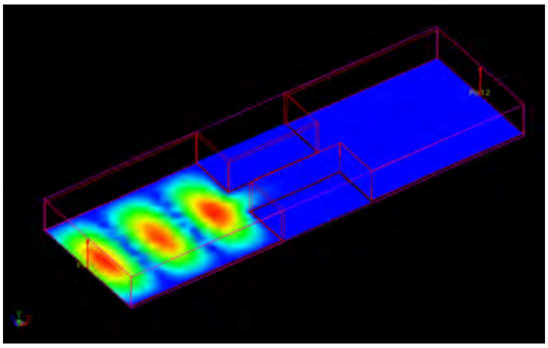
(a) S_{11} for the given waveguide. (b) S_{12} for the given waveguide.

Figure 2: E-field propagation in the given waveguide.

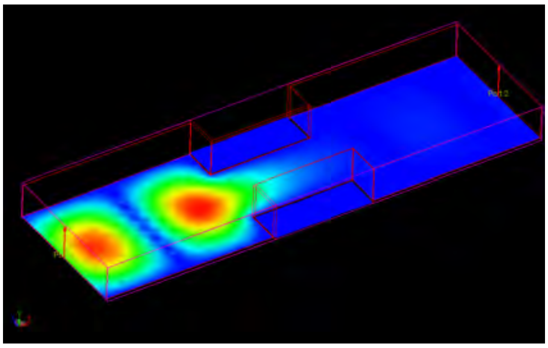
reflection with respect to frequency is shown. What one actually expect is that the curve would start at 0 dB at DC and then quickly decrease when the cutoff frequency is reached. One can de facto see that something is happening at the cutoff, but this is not that the reflection is decreasing. This could possibly be due to some kind of biasing problem in the solver. Resulting in that the level of reflection appears to be lower in the graph than it actually is. In Figure 2b the transmission is depicted as a function of frequency. As expected the S_{12} is very low until the point of f_c where complete throughput is present.

Task B

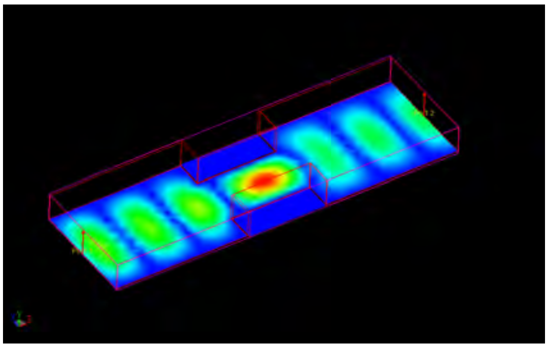
If three waveguide sections are put into union, one can alter the throughput from section one to three by for instance changing the dimensions of the middle section. In this subtask this change of dimension is going to be made in two ways. The first way that is going to be simulated is different widths of the middle section. All smaller than the width of the outer two sections. In Figure 3 the E-field for different frequencies and middle section widths is depicted. It is easy to see that with a smaller width of the middle section a higher frequency of the electromagnetic waves is needed in order for them to make it through. In Figure 3b and 3c this is very obvious. One can understand this by considering the middle section as just an other waveguide with a higher cutoff frequency. In Figure 3a the width is so small that basically nothing goes through for the simulated frequencies. The opposite occurs in Figure 3d, where the width of the middle section is comparable with the surrounding waveguides. Almost everything is let through. The conclusions can also be verified by looking at the S-parameters depicted in Figure 4.



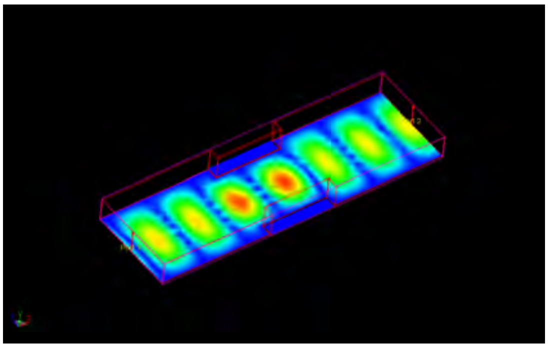
(a) Width of middle section is 10 mm, f = 9 GHz.



(b) Width of middle section is 20 mm, f = 6.2 GHz.



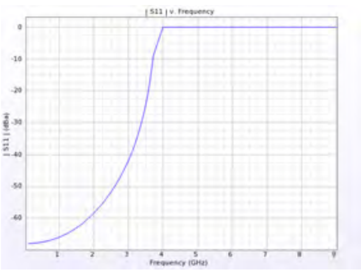
(c) Width of middle section is 20 mm, f = 9 GHz.



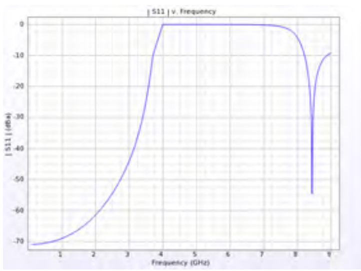
(d) Width of middle section is 30 mm, f = 9 GHz.

Figure 3: E-field propagation in the given waveguide.

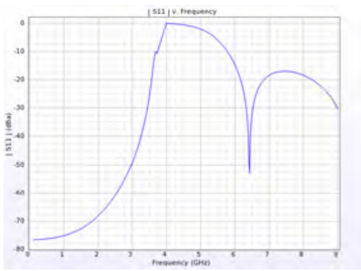
As in the first subtask the reflection coefficient start close to zero. This could mean that wave at low frequencies does not propagate nor reflects at the waveguide, but is rather dissipated in some way. One can compare the two extreme cases of Figure 4a and 4c. It is easy to see that when the frequency is above cut off for the middle section the reflection decreases. The same can of course also be seen in the S-parameter S_{12}, depicted in Figure 5.



(a) Width of middle section is 10 mm.

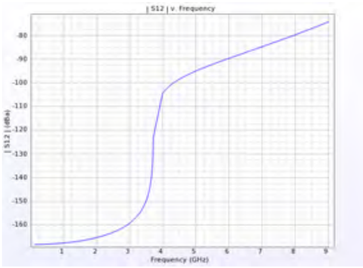


(b) Width of middle section is 20 mm.

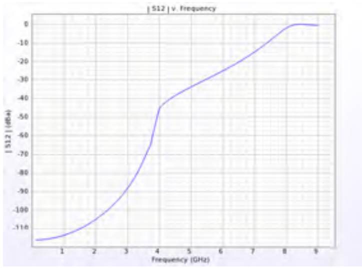


(c) Width of middle section is 30 mm.

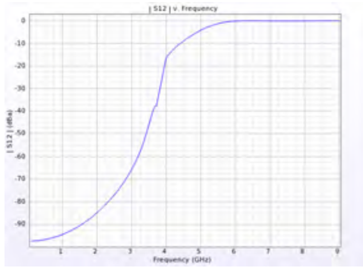
Figure 4: S_{11} for different widths of the middle section.



(a) Width of middle section is 10 mm.



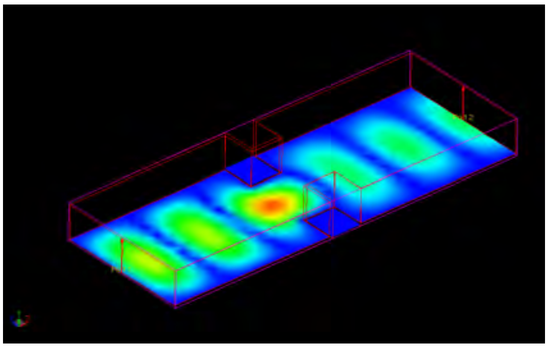
(b) Width of middle section is 20 mm.



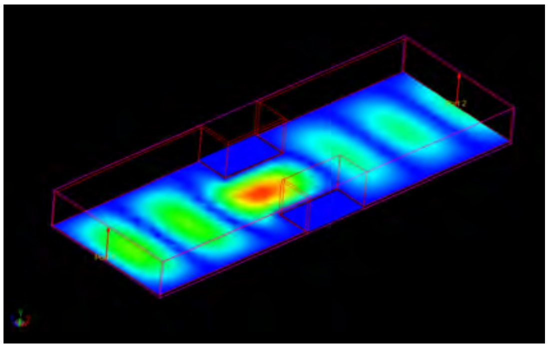
(c) Width of middle section is 30 mm.

Figure 5: S_{12} for different widths of the middle section.

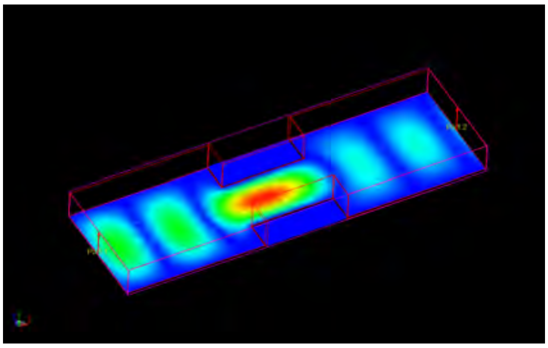
The second way the dimensions are going to be changed is by fixing the width of the middle section to be 20 mm and then alter the length of the section. The E-field of some different simulation scenarios are depicted in Figure 6. If a comparison of the pictures in Figure 6 is made, one can see that the



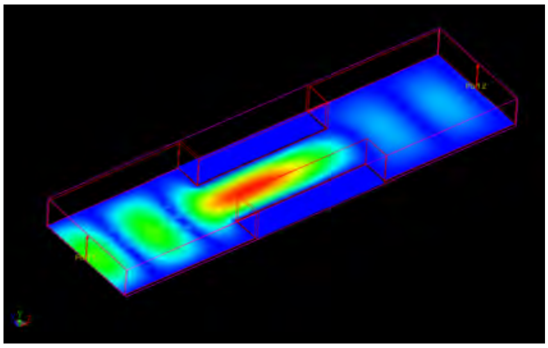
(a) Lenght of middle section is 1 cm.



(b) Lenght of middle section is 2 cm.



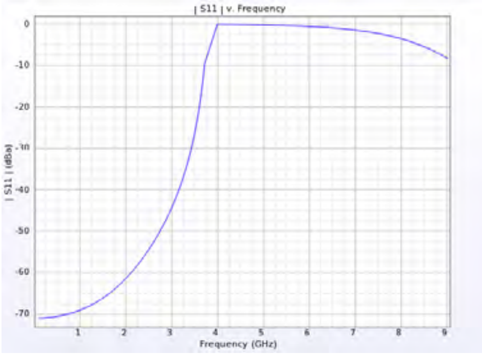
(c) Lenght of middle section is 3 cm.



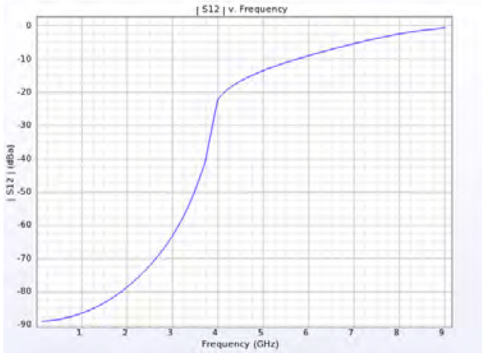
(d) Lenght of middle section is 5 cm.

Figure 6: E-field propagation in the given waveguide at about 9 GHz.

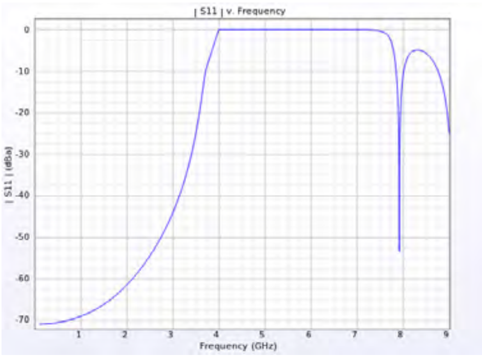
amount of E-field let through is dependent on the length of the section. A longer section gives a weaker throughput. This can also be observed in the S-parameters shown in Figure 7. It is clear that the additional section weakens the signal up to a certain frequency. This is a so called attenuator and it can be used in microwave systems to eliminate resonances.



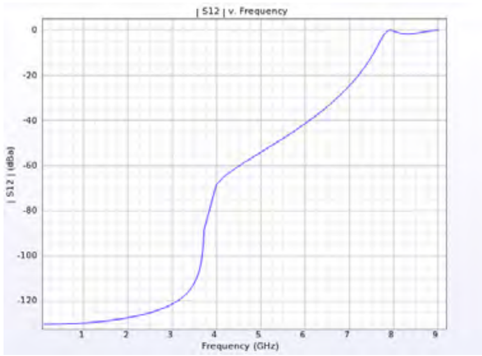
(a) S_{11} for a middle section length of 1 cm.



(b) S_{12} for a middle section length of 1 cm.



(c) S_{11} for a middle section length of 5 cm.



(d) S_{12} for a middle section length of 5 cm.

Figure 7: S_{11} and S_{12} for the shortest and longest section length simulated.

Design of a Chebyshev transformer

Here a impedance transformer of type Chebyshev is going to be simulated. The design frequency is 6 GHz. There are also some other design parameters to take in to account. The ratio between the input and the load, $\frac{Z_0}{Z_L}$, should be 4.2. The ripple level, Γ_m, should be 0.05 within the fractional bandwidth of 1. Also, the width, a, of the waveguide is kept constant at 80 mm.

The first thing one has got to do is to find out the number of sections needed for a specific design. To do this, Equation (5.63) and (5.64) in [1] can be used. Equation (5.64) gives that:

$$\frac{\Delta f}{f_0} = 2 - \frac{4\theta_m}{\pi}. \tag{2}$$

Since the fractional bandwidth, $\frac{\Delta f}{f_0}$, is 1, it is clear that $\theta_m = \frac{\pi}{4}$. Since the ratio between Z_0 and Z_L is known to be 4.2, both the sides of the equality of (5.63) can be plotted as a function of N sections in the transformer. This is depicted in Figure 8. As can be seen in the plot, the two sides of the equality intersects at $N = 3.6$. Since it is not in the scope of this exercise to deal with fractional sections, the approximation that $N = 4$ is made. Now a new value of θ_m must be calculated since N is fixed. Once again Equation (5.63) can be used, but now in the reversed manner. θ_m is calculated to be 0.24π. Next thing to do is to find the reflection coefficients for every section passing. This is done with the aid of Equation (5.61) in [1], where the function of reflection, $\Gamma(\theta)$, is set to be equal to $Ae^{-jN\theta}T_N(\sec\theta_m\cos\theta)$, where $T_N(\sec\theta_m\cos\theta)$ is the Nth Chebyshev polynomial. Since $N = 4$, the following relation is obtained:

$$2e^{-j4\theta}[\Gamma_0\cos 4\theta + \Gamma_1\cos 2\theta + \frac{1}{2}\Gamma_2] = Ae^{-jN\theta}[\sec^4\theta_m(\cos 4\theta + 4\cos 2\theta + 3) - 4\sec^2\theta_m(\cos 2\theta + 1) + 1] \tag{3}$$

By using the fact that the reflections between the sequence of sections must be symmetrical around the

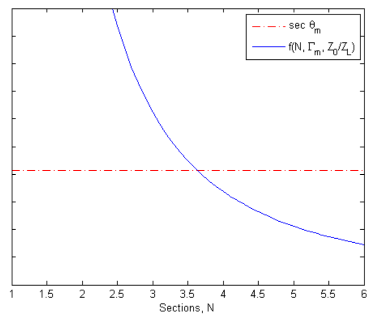


Figure 8: Plotting of Equation (5.63) in [1]. Where the curves meet, the optimum value of N is obtained. This is the case for $N = 3.6 \approx 4$

middle point one obtain the following:

$$\begin{aligned}
\Gamma_0 &= \tfrac{1}{2} A \sec^4 \theta_m \\
\Gamma_1 &= 2A \left(\sec^4 \theta_m - \sec^2 \theta_m\right) \\
\Gamma_2 &= A \left(3 \sec^4 \theta_m - 4 \sec^2 \theta_m + 1\right) \\
\Gamma_3 &= \Gamma_1 \\
\Gamma_4 &= \Gamma_0
\end{aligned}$$

Since it is given that the maximum reflection coefficient magnitude permitted in the passband, Γ_m, is 0.05, the value of A is also given since $\Gamma_m = |A|$. Though, A has got to be negative according to Equation (5.62) in [1], since $Z_0 > Z_L$. With all values as stated, the numerical values of Γ_n is calculated to be:

$$\begin{aligned}
\Gamma_0 &= -0.0889 \\
\Gamma_1 &= -0.167 \\
\Gamma_2 &= -0.206 \\
\Gamma_3 &= -0.167 \\
\Gamma_4 &= -0.0889
\end{aligned}$$

To be certain that the calculated coefficient gives the wanted $\Gamma(\theta)$ pattern, a plot of the left hand side in Equation (9) is appropriate, as depicted in Figure 9

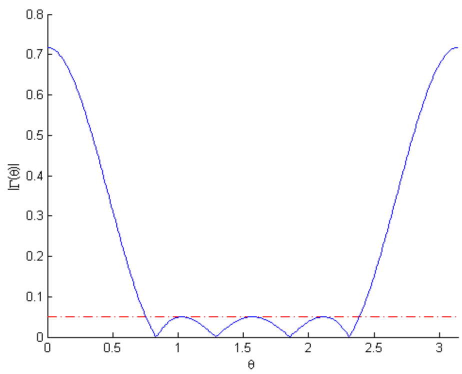


Figure 9: $|\Gamma|$ as a function of θ. The ripple floor is marked in red.

To calculate the ε_r of a section, several steps has got to be gone through. As a first step, the assumption that the wave is entering the transformer coming from a section filled with air. Since it is known that

$$Z_{\text{medium}} = \sqrt{\frac{\mu}{\varepsilon}} \tag{4}$$

and

$$Z_{\text{waveguide}} = \frac{Z_{\text{medium}}}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}, \tag{5}$$

it is easy to calculate the impedance of the following waveguide using the definition of the reflection coefficient:

$$Z_{n+1} = Z_n \left(\frac{1+\Gamma}{1-\Gamma}\right). \tag{6}$$

Once the impedance of a section is known it is easy to calculate the corresponding ε_r. This is done with help of Equations (4) and (5) along with the fact that the cut off frequency $f_{c_{10}} = \frac{1}{2a\sqrt{\mu\varepsilon}}$. The resulting expression for ε is then:

$$\varepsilon = \varepsilon_r \varepsilon_0 = \frac{\mu}{Z^2_{\text{waveguide}}} + \frac{1}{4a^2 f^2 \mu}. \tag{7}$$

When ε_r is known, it is also easy to calculate the length of a section, since one section should have the length $\frac{\lambda}{4}$ and the wavelength in a given waveguide can be expressed as [2]:

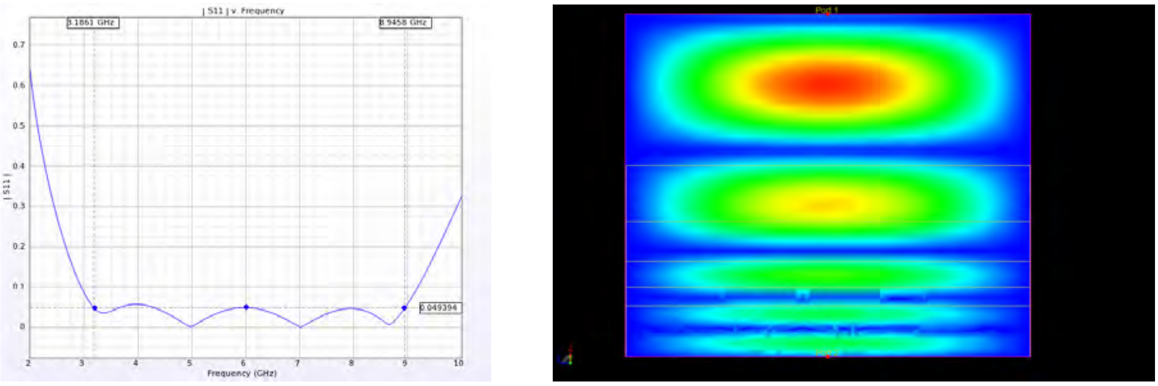
$$\lambda_{\text{waveguide}} = \frac{1}{f\sqrt{\varepsilon\mu}} \cdot \frac{1}{\sqrt{1-\left(\frac{1}{2af\sqrt{\varepsilon\mu}}\right)^2}}. \tag{8}$$

With all the above give relations one gets all the necessary parameters for simulation. The parameters are given in Table 1. When the values in Table 1 are applied to a section waveguide design in EMPro,

Table 1: The data used in the simulations for the multi dielectric Chebyshev transformer

Impedance	**Impedance value**	ε_r	**Section length**
Z_0	397 Ω	1	N/A
Z_1	332 Ω	1.38	11 mm
Z_2	238 Ω	2.60	7.9 mm
Z_3	157 Ω	5.83	5.2 mm
Z_4	113 Ω	11.5	3.7 mm
Z_L	94.4 Ω	16.4	N/A

the results in Figure 10 are obtained. As can be seen in Figure 10a the demand for the ripple factor is almost fulfilled as well as the fractional bandwidth. In Figure 10b the E-field of the transformer at the design frequency is depicted.



(a) S_{11} for the transformer.

(b) E-field in the transformer at 6 GHz.

Figure 10: The reflection parameter and E-field for the multi dielectric Chebyshev impedance transformer.

To change the design from considering different dielectrics, to deal with a change in waveguide height instead, there is a simple relation one can use:

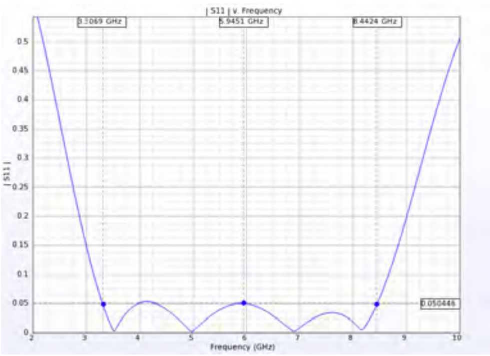
$$b_{n+1} = b_0 \frac{Z_{n+1}}{Z_0}, \tag{9}$$

where b_0 is the height of the initial waveguide and b_{n+1} is the height corresponding to the section with impedance Z_{n+1}. The heights calculated is to be found in Table 2. Every section has got the same

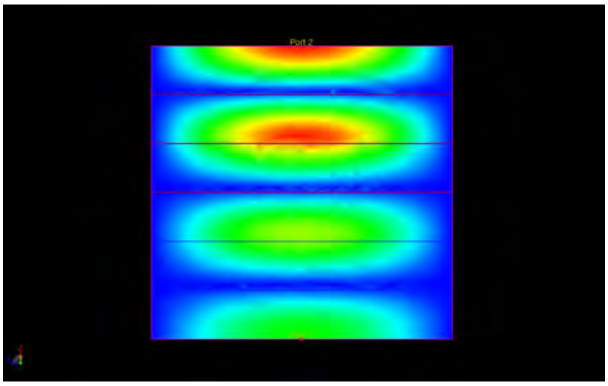
Table 2: The height data used in the simulations for the multi height Chebyshev transformer

Height	**Height value**
b_0	10 mm
b_1	8.4 mm
b_2	6.0 mm
b_3	4.0 mm
b_4	2.8 mm
b_L	2.4 mm

physical length since the medium in every section is air. The section length should be $\frac{\lambda}{4}$ and Equation (8) the section length is calculated to be 13.14 mm. The results from the simulation can be seen in Figure 11.



(a) S_{11} for the transformer.



(b) E-field in the transformer at 6 GHz.

Figure 11: The reflection parameter and E-field for the multi height Chebyshev impedance transformer.

The result in both types of transformer is quite good with respect to ripple level and fractional bandwidth. Therefore, optimisation is not necessary and would be very tricky. While trying to optimise, one can see that there is a tradeoff between ripple floor and fractional bandwidth and also how well align the transformer is around the center frequency of 6 GHz. If one were about to choose which design to go for when it comes to fabrication the choice is easy. The air filled one. It would be much simpler to fabricate and cost a lot less.

References

[1] Pozar DM. Microwave engineering. 4th ed. John Wiley & Sons, Inc., 2012.

[2] Waveguide mathematics at Microwaves101.com [cited 2012-12-06]. Available at *http://www.microwaves101.com/encyclopedia/waveguidemath.cfm*