MCC121 Microwave Engineering
Assignment: Impedance matching

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Problem 1

In this first problem the task is to match a transistor with given scattering parameters to a peripheral system with the characteristic impedance, Z_0, of 50 Ω. The system frequency is 4 GHz. The problem layout is depicted in Figure 1.

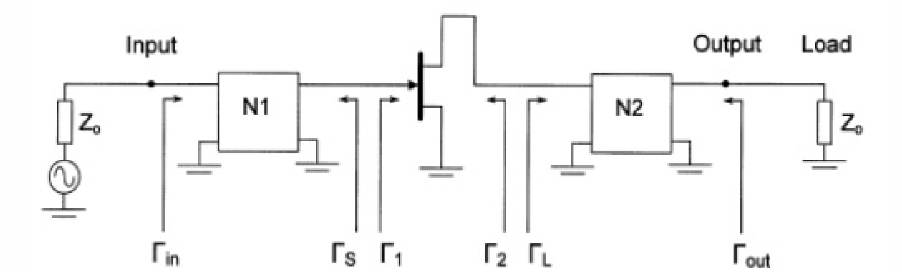


Figure 1: Layout of the network in Problem 1.

The S-parameters for the transistor is given as follows:

$$S_{11} = 0.65\angle 110^\circ \quad S_{12} = 0$$
$$S_{21} = 1.74\angle 32^\circ \quad S_{22} = 0.35\angle\text{-}70^\circ$$

It is also given that maximum power gain should be designed for. This yields the following relations:

$$\Gamma_S = S_{11}^* = 0.65\angle\text{-}110^\circ$$
$$\Gamma_L = S_{22}^* = 0.35\angle 70^\circ$$

Problem 1a

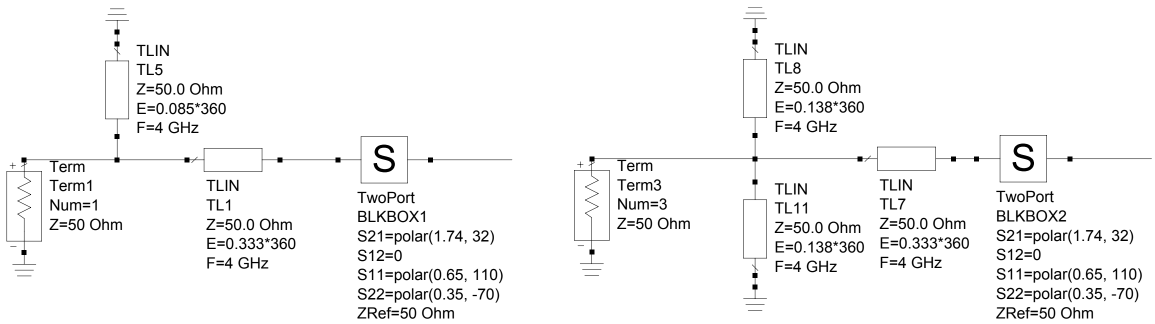
In the first sub problem it shall be shown that the input matching network N1, depicted in Figure 1, can be realised with a distributed matching network consisting of a transmission line and two balanced shunt connected stubs. To solve this, one can first design the matching network considering just one unbalanced stub which then easily can be converted into two balanced stubs. This unbalanced solution is shown in the Smith chart in Appendix A1. The solution is reached by starting in the point of the network where Γ_S is pointing towards the input. The input point is here considered to be the load. At 1 and 2 Γ_S is found in the Smith chart. Then at 3, a translation towards the load is made with a constant radius turn around the center point of no reflection until the admittance circle that cuts through the center point is intersected. At this point a normalised susceptance of -1.7j is given and to get there, a length of 0.333λ is traveled in the Smith chart. To then at 4 get down to the matched center point, one can either, in order to cancel out the -1.7j normalised susceptance, add a single unbalanced stub or two balanced stubs. If shunt stubs are of interest, the far left side of the chart is the starting point. Then one turn towards the load until the susceptance is canceled out. If one stub is used, this will be after 0.085λ. If two stubs are used one can cancel the susceptance by canceling half the value by each stub. Both stubs have in this case to cancel a normalised susceptance of -0.85j. This is achieved by turning 0.138λ on each stub.

From the literature [1] one can also find the expression:

$$l_{SB} = \frac{\lambda}{2\pi} \arctan\left(2 \tan \frac{2\pi l_S}{\lambda}\right) \tag{1}$$

where l_{SB} is the length of the balanced stubs and l_S is the length of the unbalanced stub. With the value of l_S obtained previously $l_{SB} = 0.1383\lambda$.

To validate the correctness of the results one can do an easy verification in Agilent Advanced Design System (ADS). The schematic in ADS for the input side is depicted in Figure 2. When simulating S_{11} for both circuits, the Smith chart representation in Figure 3 is obtained. As one can see, both the two solutions are adequate, but preferably should the unbalanced solution be chosen since it includes less microstrip length.



(a) The unbalanced stub matching network.

(b) The balanced stubs matching network.

Figure 2: ADS schematics of the input side.

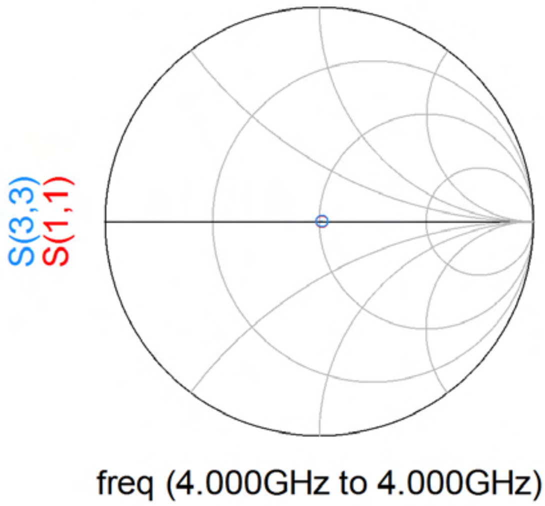
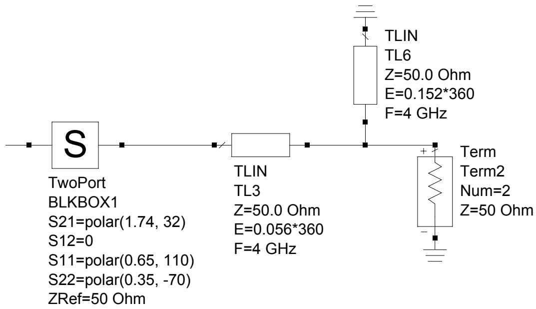


Figure 3: The reflective S-parameters for the input matching networks. The numbering is due to the termination port numbering in Figure 2. Though hard to see in the picture, the two points overlap, stating that the two solutions are theoretically equivalent.

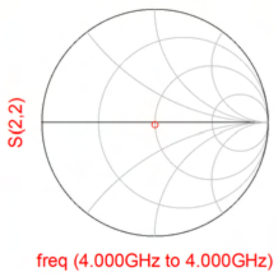
Problem 1b

The second task in Problem 1 is also to design a matching network, but now for the output side. The way to go about it is the same as in the first task, with the difference that we now start in the point where Γ_L is pointing towards N2 and then move out against the load, as depicted in Figure 1. The solution Smith chart is found in Appendix A2.

At ① and ②, Γ_L is plotted in the chart. As earlier stated, $\Gamma_L = 0.35\angle 70°$. At ③ the translation to the susceptance circle of interest is made, giving a rotation of 0.056λ towards the load. At the current point a normalised susceptance value of -$0.7j$ is present. The rotation needed to cancel this value out and thereby reaching the center point is 0.152λ. This is done at ④.To validate the solution it was simulated in ADS with the schematic in Figure 4a. The Smith chart of the simulation, depicted in Figure 3, shows that the system is almost matched to the output and the solution is therefore almost the desired one.



(a) The ADS schematic of the output.



(b) The reflective S-parameter of the output matching network. The circle is somewhat distant from the middle, but it is anticipated considering the roughness of the Smith chart solution technique.

Figure 4: Schematic and simulation result of the output matching network.

Problem 1c

To calculate the lengths and widths of all of sections one can easily use the LineCalc application in ADS. In order to get an accurate calculation of the spatial dimensions, one has to give LineCalc a set of parameters for the microstrip. Three parameters are already given in the problem description:

$$\begin{aligned}&\varepsilon_r = 2.3\\&\text{Substrate thickness}, H = 0.4 \text{ mm}\\&f = 4 \text{ GHz}\end{aligned}$$

The other parameters needed to do the calculation is taken from [2]:

$$\begin{aligned}&\text{Loss tangent}, \tan\delta = 0.0022\\&\text{Conductor thickness}, T = 35\ \mu\text{m}\\&\text{Metal conductivity (copper)}, \sigma = 5.8 \cdot 10^7 \text{ S/m}\end{aligned}$$

The width of every section will be the same since we want the same characteristic impedance in all of the system and have the same material constants everywhere. According to LineCalc the width should be 1.16 mm. To verify this, one can use the equations given in [2] to calculate the line width. In the paper there are two equations one can choose between for calculating the width, depending on the ratio between the width of the conductor and the substrate thickness. In the present case w/H is very close to the boundary between the two cases. Hence, both equations can be used. Since the only aim of this calculation is to quantitatively verify the correctness of LineCalc, the more simple equation will be used:

$$w = 4 \cdot H \left(\frac{e^A}{2} - e^{-A}\right)^{-1} \tag{2}$$

where

$$A = \frac{Z_0}{120}\sqrt{2(\varepsilon_r + 1)} + \frac{\varepsilon_r - 1}{\varepsilon_r + 1}\left(0.226 + \frac{0.121}{\varepsilon_r}\right)$$

With the parameters as earlier given the width is calculated to be 1.21 mm. This is a bit higher than calculated in LineCalc, but it is difficult to know what parameters and simplifications are used the equation, therefore a deduction why this result is achieved will be put aside. The result is at least quantitatively comparable with the result from LineCalc.

To calculate the spatial length of the sections one can easily use the results acquired from the Smith charts together with the well known relations:

$$\lambda = \frac{c}{f} \quad \text{and} \quad c = \frac{1}{\sqrt{\varepsilon\mu}}$$

To get a result even quicker, LineCalc can also be used in this case. What one put into LineCalc is the wavelength fraction times 360°, e.g. $l = 0.333\lambda \Rightarrow l_{\text{Electrical}} = 0.333 \cdot 360°$. If the values for the stubs and the lines are put in, the following result is achieved:

Line in: 17.96 mm	Stubs in: 7.442 mm
Line out: 3.010 mm	Stub out: 8.197 mm

Problem 2

In the second problem a load impedance, Z_L, of 20-80j Ω is about to be matched to a 50 Ω transmission line. The system frequency is now 5 GHz. To do this the Smith chart will be used. As always, the starting point is to normalise the load impedance with respect to 50 Ω. This gives:

$$z_L = \frac{Z_L}{50\ \Omega} = \frac{20 - 80j\ \Omega}{50\ \Omega} = 0.4 - 1.6j$$

Problem 2a

The first sub problem is to make the system matched using a distributed network consisting of a shunt stub and a quarter wave section. The Smith chart solution is to be found in Appendix A3. The first thing to do is to plot the normalised load impedance point in the chart. Thereafter, in 1, the cancellation of the $0.59j$ susceptance value is made by traveling 0.165λ away from the load, i.e. towards the generator. The real axis has now been intersected, not in the point of no reflection, but in the point with a normalised impedance value of 7. To reach the center point a quarter wave transformer can now be used at 2. Since the length is $\lambda/4$, i.e. half a complete turn in the Smith chart, a pure real impedance will be present also after the transformer. To calculate which characteristic impedance, Z_C, the transformer got to have, the very well known[3] equation for impedance transformation along a transmission line can be used:

$$Z_0 = Z_C \frac{Z_L + jZ_C \tanh(\gamma l)}{Z_C + jZ_L \tanh(\gamma l)} \tag{3}$$

Since we are not considering loss $\gamma = j\beta$. But $\beta = \frac{2\pi}{\lambda}$ and $l = \frac{\lambda}{4}$. This results in:

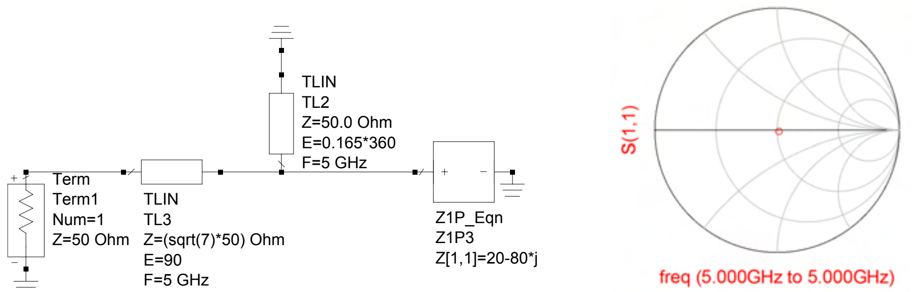
$$Z_0 = \frac{Z_C^2}{Z_L}$$

This is of course the same as:

$$Z_C = \sqrt{Z_0 Z_L} \tag{4}$$

In the present situation $Z_L = 7 \cdot Z_0$, which results in that $Z_C = Z_0\sqrt{7} = 132\ \Omega$.

To validate the solution ADS is used. In Figure 5a the schematic of the solution is depicted and in Figure 5b the Smith chart representation of it. Considering the electric lengths and impedances of the



(a) ADS schematic of the distributed matching network in the second task.

(b) The reflective S-parameter of the distributed matching network in the second task.

Figure 5: Schematic and simulation result of the distributed matching network in the second task.

stub and the quarter wave transformer the microstrip parameters can be calculated using LineCalc. This results in the following spatial parameters:

Transformer width: 0.1285 mm	Stubs width: 1.156 mm
Transformer length: 11.46 mm	Stub length: 7.116 mm

Problem 2b

In the second subtask one is to find a solution corresponding to the solution in the first subtask, but this time with lumped components in a L-network. The network can consist of combinations of inductors and capacitors. One of the four solutions that exists is found in Appendix A4.

At 1 the translation from a normalised reactance value of -1.6j to a normalised reactance value of 0.5j. The translation is made by connecting a inductor in series with the load. The movement resulted in a change of 2.1j normalised reactance. Since this is due to a inductor in series, the following relation is true:

$$2.1j \cdot Z_0 = j\omega L$$

Since $\omega = 2\pi f$, the inductance, L, is calculated to be 3.3 nH. At the current point, the desired susceptance circle is intersected at a normalised susceptance value of -1.2j. To cancel this out, a capacitor is connected in parallel to the load and the inductor at 2. Since susceptance and admittance is now of consideration, the relation between the normalised value and the actual capacitor value looks like:

$$\frac{1.2j}{Z_0} = j\omega C$$

Hence, one gets a value of the capacitance, C, that is 0.76 pF. The schematic of the described network is to be found in Figure 6 along with the schematics of the three other solutions that are obtained in a similar manner. In Figure 7 the Smith chart representation of the solutions are all shown. All solutions are close to the center point without any parameter tuning being made. One can conclude that all solutions are comparable from a matching point of view.

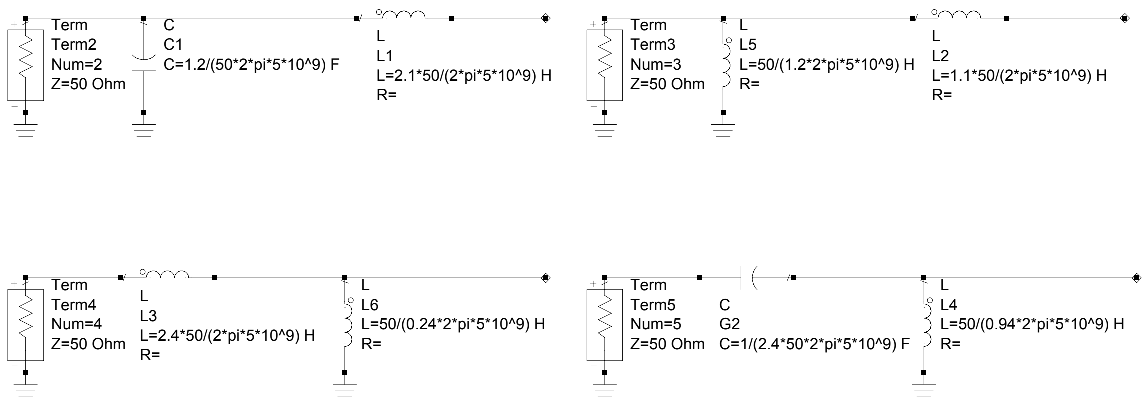


Figure 6: The four matching L-networks of interest. The circuit with termination number two is the one considered in the elaborative calculations.

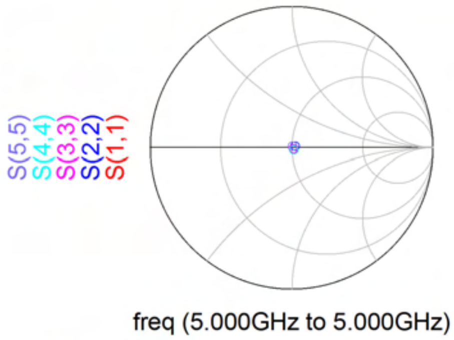


Figure 7: The Smith chart representation of the reflection parameters. The S_{11} from the previous subtask is shown for comparison.

Problem 2c

This subtask deals with the return loss (RL) of the network. RL is defined as:

$$\mathrm{RL_{dB}} = -20\log_{10}|\Gamma| \tag{5}$$

One can define this equation in ADS, replacing Γ with S_{ii} and then plot with respect to frequency. The result can be seen in Figure 8. As a general comment, it can be seen that the RL for the microstrip

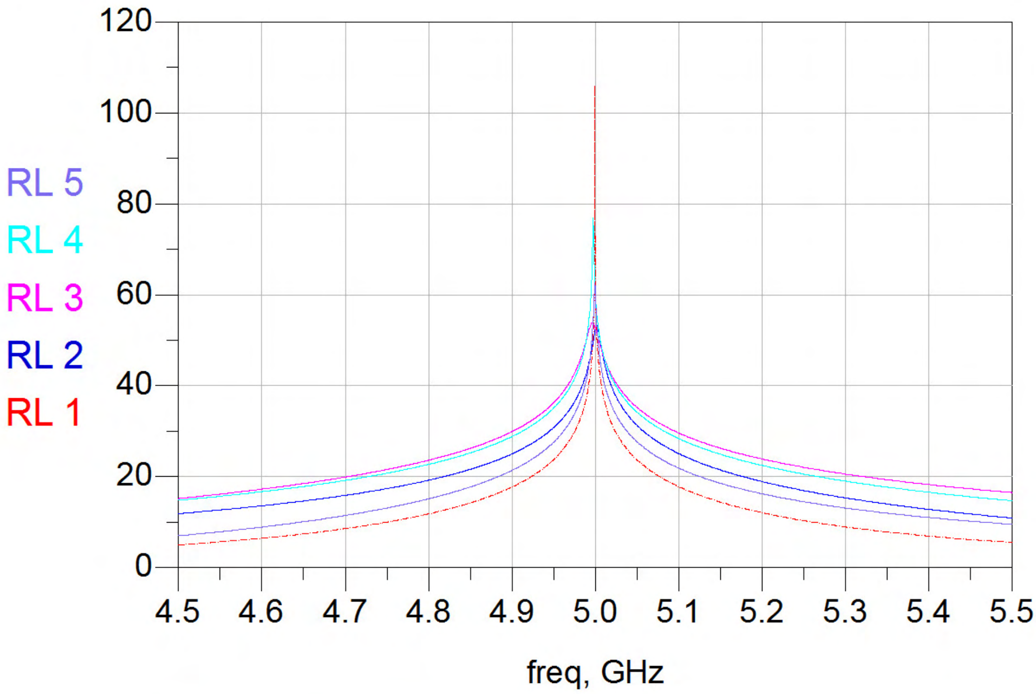


Figure 8: The RL of the five matching networks. The dashed line belongs to the distributed network.

network is much higher than for the other networks, but the bandwidth is much worse. This is easy to see if one zoom into Figure 8 as depicted in Figure 9. Of course, the RL should by definition go towards infinity when the magnitude of Γ goes towards zero, as should be the case at 5 GHz, but the simulation gives a maximum value for every topology. To calculate the bandwidth and maximum RL, there are two functions in ADS that comes in handy. The first one is the `bandwidth_func(data, level)` function, which returns the bandwidth in Hz of some given `data` at a specified `level` in dB. In this case `level` = 3 dB. The second one is the common `max(data)` function. With these two functions it is

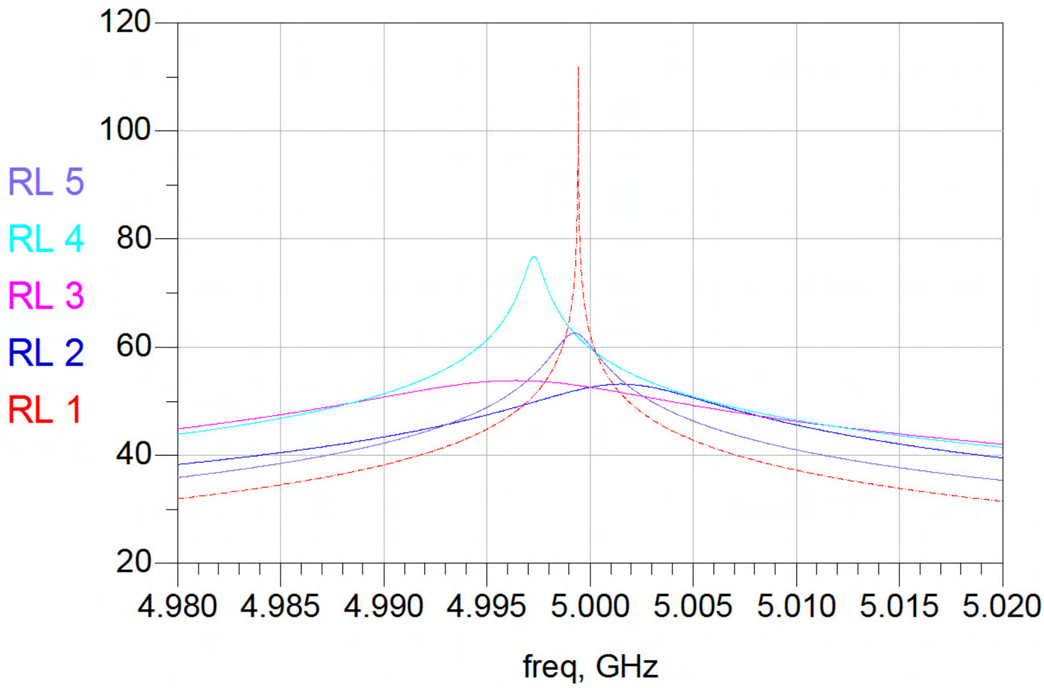


Figure 9: The RL of the five matching networks. The dashed line belongs to the distributed network.

Topology	Maximum RL (dB)	3 dB Bandwidth
Distributed	112	4.09 kHz
Lumped 1 (LC)	53.2	7.85 MHz
Lumped 2 (LL)	53.8	12.5 MHz
Lumped 3 (LL)	76.8	779 kHz
Lumped 4 (LC)	62.6	1.78 MHz

Table 1: Values of comparison for the five networks.

quite easy to compare the five networks. Table 1 contains the values of comparison. As one can see the distributed network has got a superior maximum RL, but also by far the most inferior bandwidth. The lumped networks has got better bandwidth but then also worse maximum RL. As a summary one can say that in this case it is really a trade off between RL and bandwidth.

Extra problem

The aim of this extra task is to find an equivalent π-network to a quarter wave transformer at the design frequency of 450 MHz, see Figure 10. The phase velocity in the cable is known to be 75% of the

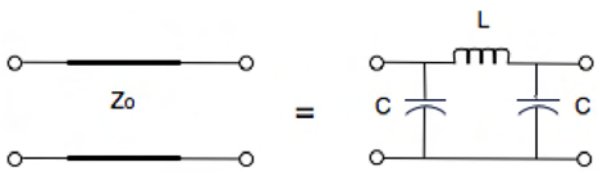
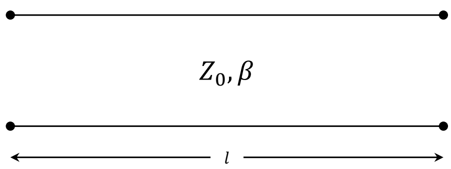
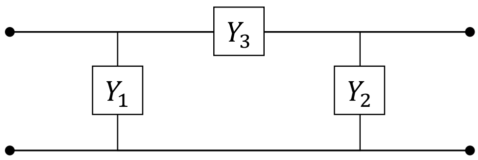


Figure 10: A depiction of the problem principle.

speed of light in vacuum. To solve this, one can use the fact that both networks can be described with



(a) Representation of a transmission line network.



(b) Representation of a π-network.

Figure 11: Network representations used when deducing $ABCD$ parameters.

$ABCD$ parameters[3]. With a network like the one in Figure 11a the following $ABCD$ parameters are present:

$$\begin{array}{ll} A = \cos\beta l & B = jZ_0 \sin\beta l \\ C = jY_0 \sin\beta l & D = \cos\beta l \end{array} \tag{6}$$

In the present case loss is not considered. Therefore it is clear that:

$$\beta = \frac{2\pi}{\lambda}, l = \frac{\lambda}{4} \Rightarrow \beta l = \frac{\pi}{2}$$

With this clear it is trivial to simplify Equations (6) into:

$$\begin{array}{ll} A = 0 & B = jZ_0 \\ C = jY_0 & D = 0 \end{array} \tag{7}$$

If a π-network as in Figure 11b is present, the $ABCD$ parameters will be as follows:

$$\begin{array}{ll} A = 1 + \frac{Y_2}{Y_3} & B = \frac{1}{Y_3} \\ C = Y_1 + Y_2 + \frac{Y_1 Y_2}{Y_3} & D = 1 + \frac{Y_1}{Y_3} \end{array} \tag{8}$$

However, the networks should be each others equvivalent. Hence, the $ABCD$ parameters in Equations (7) and (8) must be the same. The following relations are therefore obtained:

$$\begin{array}{ll} \frac{Y_2}{Y_3} = -1 & Y_3 = \frac{1}{jZ_0} \\ Y_1 + Y_2 + \frac{Y_1 Y_2}{Y_3} = jY_0 & \frac{Y_1}{Y_3} = -1 \end{array}$$

But then it is clear that:

$$Y_1 = Y_2 = -\frac{1}{jZ_0} \quad Y_3 = \frac{1}{jZ_0}$$

Which is the same as:

$$Z_{\text{Capacitor}} = Z_1 = Z_2 = -jZ_0 \quad Z_{\text{Inductor}} = Z_3 = jZ_0$$

The relations between impedance and component value is know as:

$$Z_{\text{Capacitor}} = \frac{1}{j\omega C} \quad Z_{\text{Inductor}} = j\omega L$$

Hence, the values of the components will be:

$$L = \frac{Z_0}{\omega} \quad C = \frac{1}{\omega Z_0}$$

To determine the exact value of the components, one needs to know the characteristic impedance, Z_0. Since the frequency of operation is 450 MHz it is reasonable that the cable of investigation is the RG-8X coaxial cable. The RG-8X has got a velocity factor of 75% as stated in the task description and a characteristic impedance of 50 Ω [4]. Since $Z_0 = 50\ \Omega$ and $\omega = 2\pi \cdot 450 \cdot 10^6$ rad/s the component values of interest are:

$$L = 17.7 \text{ nH} \quad C = 7.1 \text{ pF}$$

References

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